GREAT CIRCLE COMPUTATIONS

ANY CIRCLE ON the surface of a sphere which shares the diameter of the sphere is called a *great circle*. It is the largest possible circle which can be drawn on the surface. The equator and all meridians are great circles on the earth. A great circle always cuts a sphere into two equal hemispheres.

Although the earth is about 24 nautical miles out-of-round (*oblate*), it can be considered a sphere for navigational purposes. The most significant property of a great circle is that it represents the shortest distance between any two points which fall on its arc. Only one great circle passes through any two points on the earth, unless the points are *antipodal*, i.e., 180°apart on opposite sides of a diameter of the sphere. The north and south poles are *antipodal*. A long rod passed through the earth which pierced two antipodal points would pass through the center of the sphere. Antipodal points have an infinite number of great circles passing through them. All meridians are great circles. Every meridian passes through both poles.

The minimum distance property makes the great circle the ideal route for sailing or flying great distances. The difficulty in doing so depends upon still another property of great circles. They intersect meridians at varying angles. The meridians are always oriented north-south through the geographic poles. The angle which a course line makes with a meridian is the basis for computing the bearing to be followed by a craft. When navigating great circle routes constant changes of course are necessary to stay on the great circle. Wherby, with rhumbline navigation one course is steered at a constant angle even though the distance travelled may be much greater.

Actually most great circle navigation is not done along the actual arc of the circle but rather along a series of chords which closely approximate the arc of the circle. These chords are actually rhumblines. So it is possible to steer several rhumblines of constant course, changing course with each new chord and come pretty close to following the route required by a circle.

A further difficulty for great circle navigation is the location of the *vertex* of the circle. This is the point furthest north or south along the great circle track. It may take ships into polar regions where severe storms and dangerous ice floes may cripple a ship. It may even cross land which poses obvious difficulties for watercraft. Although not such a serious problem for aircraft, they cannot look upon vertex computations with impunity. Travel along a great circle route could take aircraft over sovereign, perhaps hostile, territory. So great circles cannot be followed blindly.

The mathematics of navigating on a sphere involves solution of *spherical triangles*. These are triangles on the surface of the sphere whose sides are the arcs of great circles. These great circles show up as straight lines on the following charts: transverse mercator, azimuthal equidistant, polar stereographic, Lambert conformal, polar gnomonic, oblique gnomonic and equatorial gnomonic. Great circle navigation with these charts is more convenient.

A great circle crosses a meridian at higher latitudes as the actual distance between the meridians diminishes. The difference between rhumbline distance and great circle distance increases; (1) as the latitude increases; (2) as the difference in latitude between two points decreases; or (3) as the difference in longitude increases. Rhumbline distances and great circle distances are approximately the same when distances are short—about 500 miles or less—or when the great circle is the equator or a meridian. The equator and all meridians are the only great circles which are also rhumblines.

Every great circle bisects every other great circle, so that half is in the northern hemisphere and half is in the southern. In each half there is a lowest (southern) and highest (northern) point called the *vertex*. Every great circle has two vertices, one in each hemisphere. The vertices are always antipodal. At this point the circle is tangent to a parallel of latitude and is truly east-west in its orientation at a bearing of 90° or 270°. The circle is beginning to turn north if it was going south and south if it was going north. As the circle crosses the equator, the angle at which it intersects the equator is equal to the latitude of the vertex.

In traveling from New York to London an aircraft would steer 50° on the New York meridian. This would increase to 90° over Iceland which is further north than New York. The angle would continue to increase as the aircraft approached London.

The program below will calculate the distance and initial true course for great circle navigation. The true course must necessarily change. The more rapid the change the truer the circle. It can be recomputed by allowing the program to calculate intermediate points on the great circle and then reinputing those points as new starting points for a smaller bite on the great circle. The program will compute a vertex. The vertex may not always be between the start and destination which you have selected along the arc of the great circle. It's possible to sail from New York to Nova Scotia along a great circle whose vertex lies in Iceland.

Any number of intermediate points can be computed. They will be computed according to the longitude which you select. These will be points which actually lie along the great circle path. They can serve as endpoints for rhumbline chords actually flown to approximate the circle.

The following equations were used:

$$D = 60 \cos^{-1}[\sin L_s \sin L_d + \cos L_s \cos L_d \cos(\lambda_d - \lambda_s)]$$

$$T = \cos^{-1}[(\sin L_d - \sin L_s \cos(D/60))/\sin(D/60) \cos L_s)]$$

unless $\sin(\lambda_d - \lambda_s) < 0$, then $T = 360° - T$

$$L_i = \tan^{-1}(A/B), \text{ where}$$

$$A = \tan L_d \sin(\lambda_i - \lambda_s) - \tan L_s \sin(\lambda_i - \lambda_d)$$

and,

$$B = \sin(\lambda_d - \lambda_s)$$

$$\lambda_v = \lambda_s - \tan^{-1}[1/(\tan T \sin L_s)]$$

where,

- D = the great circle distance
- T = the *initial* true course
- L_s = the latitude of the start
- L_d = the latitude of the destination
- L_i = the latitude of an intermediate point
- $\lambda_s, \lambda_d, \lambda_i$ = longitude of start, destination or intermediate points
- λ_v = the longitude of the vertex

To compute L_v, the latitude of the vertex, find λ_v and plug it into L_i in place of λ_i. This is done by the program on line 490.

If the vertex is at (L_v, λ_v) then the antipodal vertex will be at $(-L_v, \lambda_v + 180°)$. The great circle will then cross the equator at $\lambda_v + 90°$ and $\lambda_v - 90°$.

Much of the complexity of the program is involved with converting degrees, minutes and seconds to decimal degrees and then to radians and back again.

The use of the INP function in place of the more common INT is explained in detail in the section on SPHERICAL TRIANGLES.